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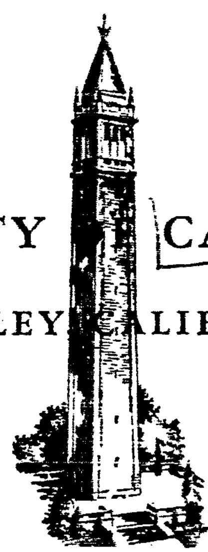

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t: PLASMAS IN SPACE:

UNSTABLE LONGITUDINAL PLASMA OCCILLATIONS,

by

Elisabeth A. Cooper and Ann Palm 2 Mar. 1964 47p refs

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Abstract. The linear theory of growing longitudinal plasma oscillations is reviewed here because of its importance in explaining ionospheric, interplanetary, and solar phenomena. Dispersion relations are derived for such oscillations in cold and warm collisionless plasmas assumed to be homogeneous in space. Several velocity distributions are considered both without and with an external magnetic field. The physical mechanism of growth is discussed, and finally the natural phenomena are described which today are attributed to this type of plasma instability. Author

1. Introduction

This review is concerned with the growth of small-amplitude longitudinal oscillations in plasmas and with the occurrence of such plasma instabilities in space. Although nuclear fusion projects have provided the stimulus for much research in plasma theory, there is now an increasing awareness of the relevance of this theory to plasmas in the upper atmosphere of the earth and in interplanetary space for which new applications of laboratory plasma models are being discovered.

Unstable longitudinal oscillations were first detected in two cold streams of charged particles moving at different velocities (PIERCE, 1948; HAEFF, 1949). This was called the "two-stream instability" and the term has since been used in a more general sense, applied to multistream interactions and to the instability which appears when the electrons in a plasma have a net drift relative to the ions. The term "drift instability" is also used in the latter case.

In Section 2 of this review dispersion relations are derived by linear theory for longitudinal oscillations in cold and warm homogeneous collisionless plasmas. In Section 3 the solutions of the dispersion relations are obtained for various plasma configurations, and the attempts which have been made to remove some of the restrictions of the theory are discussed. The physical mechanisms causing the instabilities are described in Section 4, and the interpretation of phenomena such as solar radio noise in terms of plasma oscillations is presented in Section 5 which is intended to provide examples rather than a comprehensive summary.

2. Derivation of the Dispersion Relations

Two approaches may be used to examine longitudinal oscillations in plasmas. The first is based on the hydromagnetic equations which describe the conservation of density, momentum, and energy, while the second is the kinetic approach, based on the Boltzmann equation for each species of particle. Maxwell's equations connect the electromagnetic field components to the charge and current densities, but for studies of longitudinal oscillations it is often sufficient to use Poisson's equation. For zero-temperature plasmas both theories give the same result, but for plasmas possessing finite temperatures a kinetic analysis must be used in order to take account of Landau damping. LANDAU (1946) has shown that small-amplitude longitudinal oscillations in a collisionless plasma are damped. This phenomenon is discussed further in Section 4.

We solve here the initial value problem for the behavior of a nonsingular plasma system perturbed at $t = 0$, using the Laplace transform technique according to LANDAU. VAN KAMPEN (1955) treated the problem in terms of normal modes, and CASE (1959) and BACKUS (1960) performed comparative studies of the two methods. Both give the same results for stable plasmas, but according to BACKUS a normal mode analysis may lead to incorrect results for unstable plasmas.

In this section, dispersion relations are derived for small-amplitude longitudinal oscillations in a cold plasma, a warm plasma, and a plasma in an externally produced magnetic field; the plasma is assumed to be collisionless and homogeneous in space. The results are obtained by linearization of the equations describing the fluctuations and are therefore sufficient to determine whether the plasma is unstable, but insufficient to describe the final state of the plasma.

2.1 Dispersion relation for cold streams

We consider first a plasma consisting of n streams such that each stream contains only one species of particle, and that particles move only with the velocity of the stream and have no thermal motion. It is assumed that no collisions occur and that the only forces acting on the particles are internal electromagnetic. The equations of conservation of density and momentum for each stream are

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \underline{u}_j) = 0 \tag{2.1}$$

$$\frac{\partial \underline{u}_j}{\partial t} + (\underline{u}_j \cdot \nabla)\underline{u}_j - \frac{q_j}{m_j}\left(\underline{E} + \frac{\underline{u}_j \times \underline{B}}{c}\right) = 0, \tag{2.2}$$

where $\underline{u}_j$, n_j, q_j and m_j are the stream velocity and density and the particle charge and mass, respectively. $\underline{E}$ and $\underline{B}$ are the internal electric field and magnetic induction which satisfy Maxwell's equations

$$\nabla \cdot \underline{E} = 4\pi\rho \tag{2.3}$$

$$\nabla \cdot \underline{B} = 0 \tag{2.4}$$

$$\nabla \times \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} \tag{2.5}$$

$$\nabla \times \underline{B} = \frac{4\pi}{c}\underline{j} + \frac{1}{c}\frac{\partial \underline{E}}{\partial t}, \tag{2.6}$$

where

$$\rho = \sum_{j=1}^{n} q_j n_j$$

$$\underline{j} = \sum_{j=1}^{n} q_j n_j \underline{u}_j \quad . \tag{2.7}$$

The plasma is assumed to be slightly perturbed from its zero-order configuration in which $\underline{E}_o \equiv \underline{B}_o \equiv 0$, and $\rho_o \equiv 0$. If $\underline{j}_o \neq 0$, the magnetic field produced by $\underline{j}_o$ is neglected.

Substituting

$$\begin{aligned} n_j &= n_{jo} + n_{j1}(\underline{r},t) \\ \underline{u}_j &= \underline{u}_{jo} + \underline{u}_{j1}(\underline{r},t) \end{aligned} \tag{2.8}$$

into equations (2.1) and (2.2), and linearizing by including only first-order terms in the perturbed quantities, we obtain

$$\frac{\partial n_{j1}}{\partial t} + n_{jo}(\nabla \cdot \underline{u}_{j1}) + \underline{u}_{jo} \cdot \nabla n_{j1} = 0 \tag{2.9}$$

$$\frac{\partial \underline{u}_{j1}}{\partial t} + (\underline{u}_{jo} \cdot \nabla)\underline{u}_{j1} - \frac{q_j}{m_j}\left(\underline{E}_1 + \frac{\underline{u}_{jo} \times \underline{B}_1}{c}\right) = 0. \tag{2.10}$$

Next we take Fourier-Laplace transforms in space and time

$$F(\underline{k},\omega) = \int_{-\infty}^{\infty} d^3r \int_0^{\infty} dt F(\underline{r},t) e^{\,i(\omega t - \underline{k} \cdot \underline{r})} ,$$

where $|F(\underline{r},t)| < |Me^{\gamma t}|$ and $\mathrm{Im}(\omega) > \gamma$, and obtain from equations (2.9) and (2.10)

$$\underline{u}_{j1}(\underline{k},\omega) = \frac{\left[\frac{q_j}{m_j}\left(\underline{E}_1 + \frac{\underline{u}_{jo} \times \underline{B}_1}{c}\right) + \underline{u}_{j1}^{*}\right]}{i(\underline{k} \cdot \underline{u}_{jo} - \omega)} , \tag{2.11}$$

and

$$n_{j1}(\underline{k},\omega) = \frac{n_{j1}^{*}}{i(\underline{k} \cdot \underline{u}_{jo} - \omega)} - \frac{n_{jo}\underline{k} \cdot \left[\frac{q_j}{m_j}\left(\underline{E}_1 + \frac{\underline{u}_{jo} \times \underline{B}_1}{c}\right) + \underline{u}_{j1}^{*}\right]}{i(\underline{k} \cdot \underline{u}_{jo} - \omega)^2}, \quad (2.12)$$

where n_{j1}^{*} and $\underline{u}_{j1}^{*}$ are the Fourier transforms of $n_{j1}(\underline{r},t = 0)$ and $\underline{u}_{j1}(\underline{r},t = 0)$, respectively. Define $\underline{E}_1^{*}$ and $\underline{B}_1^{*}$ similarly.

The Fourier-Laplace transforms of Maxwell's equations (2.3) to (2.6) are

$$i\underline{k} \cdot \underline{E}_1 = 4\pi\rho_1 \quad (2.13)$$

$$\underline{k} \cdot \underline{B}_1 = 0 \quad (2.14)$$

$$i\underline{k} \times \underline{E}_1 = \frac{1}{c}\,(i\omega\underline{B}_1 + \underline{B}_1^{*}) \quad (2.15)$$

$$i\underline{k} \times \underline{B}_1 = \frac{4\pi}{c}\,\underline{j}_1 - \frac{1}{c}\,(i\omega\underline{E}_1 + \underline{E}_1^{*}) \quad . \quad (2.16)$$

When no external magnetic field is present ($\underline{B}_o = 0$), a consistent solution of equations (2.11) to (2.16) can be found for which $\underline{B}_1 = 0$ and $\underline{E}_1$ is parallel to $\underline{k}$. This solution represents pure longitudinal oscillations. Combination of equations (2.7), (2.12), and (2.13) gives the electric field for this solution

$$\underline{E}_1(\underline{k},\omega) = \left\{ \frac{-4\pi \sum_j \left[\frac{q_j n_{j1}^{*}}{i(\underline{k} \cdot \underline{u}_{jo} - \omega)} + \frac{q_j n_{jo}\underline{k} \cdot \underline{u}_{j1}^{*}}{(\underline{k} \cdot \underline{u}_{jo} - \omega)^2}\right]}{1 - \sum_j \frac{\omega_{pj}^{2}}{(\underline{k} \cdot \underline{u}_{jo} - \omega)^2}} \right\} \frac{\underline{k}}{k^2} \quad , \quad (2.17)$$

where the plasma frequency of a stream ω_{pj} is defined by

$$\omega_{pj}^2 = \frac{4\pi q_j^2 n_{jo}}{m_j} .$$

Now

$$\underline{E}_1(\underline{k},t) = \int_C d\omega\ \underline{E}(\underline{k},\omega)e^{-i\omega t}, \tag{2.18}$$

where the contour C passes above all singularities in the complex ω-plane ($\mathrm{Im}(\omega) > \gamma$). This integral can be evaluated by closure of the contour with a semicircle in the lower half of the ω-plane. The integral may then be expressed as the sum of the residues at the poles of the integrand, each pole of $\underline{E}(\underline{k},\omega)$ contributing a solution $e^{-i\omega_a t}$. Poles below the real ω-axis correspond to damped solutions, and poles above the real axis to growing solutions. Although $\underline{E}(\underline{k},\omega)$ was defined only for $\mathrm{Im}(\omega) > \gamma$, equation (2.17) defines its analytic continuation for $\mathrm{Im}(\omega) \leqq \gamma$. For nonsingular initial perturbations n_{j1}^* and $\underline{u}_{j1}^*$ the poles of $\underline{E}(\underline{k},\omega)$ are given by

$$H(\underline{k},\omega) \equiv 1 - \sum_j \frac{\omega_{pj}^2}{(\underline{k} \cdot \underline{u}_{jo} - \omega)^2} = 0. \tag{2.19}$$

This is the dispersion relation; it is examined for growing solutions in Section 3.1.

2.2 Dispersion relation for warm streams

We next consider streams of plasma in which the particles have a velocity spread about the mean stream velocity. The external fields are assumed to be zero. It is now necessary to use the Boltzmann equation rather than the hydromagnetic equations. Let $f_j(\underline{r},\underline{v},t)$ be the number density of particles of the jth stream in phase space. The collisionless Boltzmann equation, usually called the Vlasov equation, is

$$\frac{\partial f_j}{\partial t} + \underline{v} \cdot \frac{\partial f_j}{\partial \underline{r}} + \frac{q_j}{m_j}\left(\underline{E} + \frac{\underline{v} \times \underline{B}}{c}\right) \cdot \frac{\partial f_j}{\partial \underline{v}} = 0. \tag{2.20}$$

The charge and current densities in terms of f_j are

$$\begin{aligned} \rho &= \sum_j q_j \int f_j d^3v \\ \underline{j} &= \sum_j q_j \int \underline{v} f_j d^3v, \end{aligned} \tag{2.21}$$

and the components of the electromagnetic field satisfy Maxwell's equations (2.3) to (2.6). As in the preceding section, pure longitudinal oscillations are possible, and we may therefore set $\underline{B} \equiv 0$ and use Poisson's equation (2.3) for the electric field.

Consider a small departure from equilibrium

$$f_j(\underline{r},\underline{v},t) = f_{jo}(\underline{v}) + f_{j1}(\underline{r},\underline{v},t).$$

If only first-order terms are retained, equation (2.20) becomes

$$\frac{\partial f_{j1}}{\partial t} + \underline{v} \cdot \frac{\partial f_{j1}}{\partial \underline{r}} + \frac{q_j}{m_j}\underline{E}_1 \cdot \frac{\partial f_{jo}}{\partial \underline{v}} = 0.$$

Taking Fourier-Laplace transforms in space and time we obtain

$$f_{j1}(\underline{k},\underline{v},\omega) = \frac{\frac{q_j}{m_j}\underline{E}_1 \cdot \frac{\partial f_{jo}}{\partial \underline{v}} + f_{j1}^{\;*}}{i(\omega - \underline{k} \cdot \underline{v})} \tag{2.22}$$

and combination of equations (2.13), (2.21), and (2.22) gives

$$\underline{E}_1(\underline{k},\omega) = \left[\frac{-\sum_j \left(4\pi q_j \int \frac{f_{j1}^*}{\omega - \underline{k} \cdot \underline{v}} \, d^3v \right)}{k^2 + \sum_j \left(\frac{4\pi q_j^2}{m_j} \int \frac{\underline{k} \cdot \frac{\partial f_{jo}}{\partial \underline{v}}}{\omega - \underline{k} \cdot \underline{v}} \, d^3v \right)} \right] \underline{k} \ . \tag{2.23}$$

$\underline{E}(\underline{k},t)$ may now be found by inverse Laplace transformation and the residue theorem. We define $\underline{E}(\underline{k},\omega)$ for $Im(\omega) \leqq \gamma$ to be the analytical continuation of equation (2.23). The dispersion relation, which gives the poles of $\underline{E}(\underline{k},\omega)$ and therefore solutions $e^{-i\omega_a t}$, is

$$H(\underline{k},\omega) \equiv 1 + \sum_j \frac{4\pi q_j^2}{m_j k^2} \int \frac{\underline{k} \cdot \frac{\partial f_{jo}}{\partial \underline{v}}}{\omega - \underline{k} \cdot \underline{v}} \, d^3v = 0. \tag{2.24}$$

Let $u = \frac{\underline{k} \cdot \underline{v}}{k}$ and integrate f_{jo} over the components of $\underline{v}$ perpendicular to $\underline{k}$. Then

$$H(\underline{k},\omega) \equiv 1 + \frac{1}{k^2} \int_C \frac{g'(u)du}{u - \omega/k} = 0, \tag{2.25}$$

where

$$g(u) = \sum_j \frac{4\pi q_j^2}{m_j} \int d^2v_\perp f_{jo}(\underline{v}).$$

For $Im(\omega) > 0$ the contour C lies along the real u-axis from $-\infty$ to ∞. For $Im(\omega) \leqq 0$ the contour C passes below the pole of the integrand at $u = \omega/k$, and the integral is thus continuous in the ω-plane. This prescription for the analytic continuation of $\int \frac{g'(u)du}{u - \omega/k}$ was introduced by LANDAU (1946).

We now evaluate $H(\underline{k},\omega)$ for some simple distribution functions

a) $$g(u) = \sum_j \omega_{pj}^2 \; \frac{\Delta_j}{\pi} \frac{1}{(u - U_j)^2 + \Delta_j^2} \quad .$$

This is a sum of one-dimensional Cauchy resonance distributions, each distribution having a mean velocity U_j and a mean thermal speed Δ_j. The distribution is "unphysical" in the sense that the kinetic energy associated with it is infinite.

From (2.25) we obtain by elementary integration

$$H(\underline{k},\omega) \equiv 1 - \sum_j \frac{\omega_{pj}^2}{(\omega - \underline{k} \cdot \underline{U}_j + ik\Delta_j)^2} = 0. \qquad (2.26)$$

b) $$f_{jo}(\underline{v}) = \frac{4N_j\Delta_j^3}{\pi^2} \left[\frac{1}{(\underline{v} - \underline{U}_j)^2 + \Delta_j^2}\right]^3 .$$

This is a three-dimensional Cauchy resonance distribution with mean velocity $\underline{U}_j$ and mean thermal speed Δ_j. The kinetic energy associated with it is finite. From (2.25) we obtain by elementary integration

$$H(\underline{k},\omega) \equiv 1 - \sum_j \omega_{pj}^2 \frac{(\omega - \underline{k} \cdot \underline{U}_j + 3ik\Delta_j)}{(\omega - \underline{k} \cdot \underline{U}_j + ik\Delta_j)^3} = 0. \qquad (2.27)$$

c) $$f_{jo}(\underline{v}) = N_j \left(\frac{\gamma_j}{\pi}\right)^{3/2} \exp\,[-\,\gamma_j(\underline{v} - \underline{U}_j)^2] \; .$$

This is a three-dimensional displaced Maxwellian distribution with a mean velocity $\underline{U}_j$. The mean thermal speed is

$$\Delta_j = (2\gamma_j)^{-1/2} \quad ,$$

and the temperature T_j of the distribution is

$$T_j = \frac{m_j}{2\kappa\gamma_j} = \frac{m_j}{\kappa}\Delta_j^{\,2} \quad ,$$

where κ is Boltzmann's constant. For this distribution the dispersion relation from (2.25) is

$$H(\underline{k},\omega) \equiv 1 + \sum_j \frac{[1 + \zeta_j\, Z(\zeta_j)]}{k^2 D_j^{\,2}} \quad , \tag{2.28}$$

where

$$D_j = \left(\frac{\kappa T_j}{4\pi N_j q_j^{\,2}}\right)^{1/2} \quad ,$$

$$\zeta_j = \gamma_j^{\,1/2}\, (\omega - \underline{k} \cdot \underline{U}_j)/k$$

and

$$Z(\zeta) = 2i\, \exp(-\zeta^2) \int_{-\infty}^{i\zeta} \exp(-t^2)dt \quad .$$

The function $Z(\zeta)$ and its first derivative are tabulated by FRIED and CONTE (1961).

2.3 Magnetized plasma

In this section a plasma in a uniform, external magnetic field $\underline{B}_o$ is examined. Since the analysis takes the same form as that in the preceding section but is algebraically more complicated, only an outline is presented.

The basic equations are Maxwell's equations (2.3) to (2.6) and the Vlasov equation (2.20). For a small departure from equilibrium

$$f_j(\underline{r},\underline{v},t) = f_{jo}(\underline{v}) + f_{j1}(\underline{r},\underline{v},t) ,$$

the zero-order Vlasov equation becomes

$$(\underline{v} \times \underline{B}_o) \cdot \frac{\partial f_{jo}}{\partial \underline{v}} = 0 , \tag{2.29}$$

while the first-order equation is

$$\frac{\partial f_{j1}}{\partial t} + \underline{v} \cdot \frac{\partial f_{j1}}{\partial \underline{r}} + \frac{q_j}{m_j} \frac{(\underline{v} \times \underline{B}_o)}{c} \cdot \frac{\partial f_{j1}}{\partial \underline{v}} + \frac{q_j}{m_j} \left(\underline{E}_1 + \frac{\underline{v} \times \underline{B}_1}{c}\right) \cdot \frac{\partial f_{jo}}{\partial \underline{v}} = 0 . \tag{2.30}$$

Let $\underline{v} = (v_x, v_y, v_z)$ in Cartesian coordinates where $\underline{B}_o$ is taken along the z-axis. In cylindrical coordinates $\underline{v} = (v_\perp, \varphi, v_z)$, where $v_\perp{}^2 = v_x{}^2 + v_y{}^2$ and $\tan \varphi = v_y/v_x$. Then equation (2.29) is satisfied by

$$f_{jo}(\underline{v}) = f_{jo}(v_\perp, v_z) . \tag{2.31}$$

Equation (2.30) may be reduced by Fourier-Laplace transformation to an ordinary differential equation for $f_1(\underline{k},\underline{v},\omega)$, in which the only derivative of f_1 to appear is $\partial f_1/\partial\varphi$. The solution of this equation may be substituted into Maxwell's equations (2.13) to (2.16), and an equation obtained for $\underline{E}(\underline{k},\omega)$. This analysis has been performed in detail by BERNSTEIN (1958) for a stationary Maxwellian plasma, and by HARRIS (1961) for any f_{jo} satisfying equation (2.31). In general, longitudinal modes

($\underline{E}_1$ parallel to $\underline{k}$, $\underline{B}_1 \equiv 0$) and transverse modes ($\underline{E}_1$, $\underline{B}_1$ perpendicular to $\underline{k}$) cannot be propagated independently in a magnetized plasma unless $\underline{k}$ is parallel to $\underline{B}_o$. However the coupling between the modes may be neglected if $\omega/kc \ll 1$ and $v_t/c \ll 1$, where v_t is a mean thermal velocity. Under these assumptions the general dispersion relation for longitudinal oscillations with wave-vector $\underline{k}$ in the x-z plane is

$$H(\underline{k},\omega) \equiv k^2 - \sum_j \frac{4\pi i q_j^2}{m_j \omega_{cj}} \int_{-\infty}^{\infty} dv_z \int_0^{\infty} v_\perp dv_\perp \int_0^{2\pi} d\varphi \int_{\lambda\infty}^{\varphi} d\varphi' \cdot G(\varphi,\varphi')\underline{k} \cdot \frac{\partial f_{jo}}{\partial \underline{v}'} = 0 \quad , (2.32)$$

where $\omega_{cj} = \frac{q_j B_o}{m_j c}$ (the gyrofrequency), λ = sign q_j, $\underline{v}' = (v_\perp,\varphi',v_z)$ and

$$G(\varphi,\varphi') = \exp\left[-\frac{i(\omega - k_z v_z)}{\omega_{cj}} (\varphi - \varphi') + \frac{ik_x v_\perp}{\omega_{cj}} (\sin\varphi - \sin\varphi')\right]. (2.33)$$

For a distribution function of the form

$$f_{jo}(v_\perp,v_z) = N_j(\gamma_j/\pi)^{3/2} \exp\left\{-\gamma_j\left[(v_z - U_j)^2 + v_\perp^2\right]\right\}$$

which is a displaced Maxwellian, representing a stream with mean velocity U_j parallel to the magnetic field, the dispersion relation (2.32) may be reduced to

$$H(\underline{k},\omega) \equiv k^2 + 2\sum_j \gamma_j \omega_{pj}^2 \,[1 + iy_j\, I(y_j)] = 0 \;, \qquad (2.34)$$

where

$$I(y) = \int_0^{\infty} dt \, \exp\left(iyt - \frac{1}{4}\frac{k_z{}^2 t^2}{\gamma_j \omega_{cj}{}^2} - \frac{k_x{}^2 \sin^2 \frac{1}{2} t}{\gamma_j \omega_{cj}{}^2} \right) \qquad (2.35)$$

and

$$y_j = \frac{\omega - k_z U_j}{|\omega_{cj}|} \quad .$$

The function $I(y)$ is known as the Gordeyev integral (GORDEYEV, 1952).

For the general case

$$f_{jo}(\underline{v}) = f_{jo}(v_{\perp}, v_z)$$

HARRIS reduces (2.32) to the form

$$H(\underline{k},\omega) \equiv k^2 - \sum_j \frac{4\pi q_j{}^2}{m_j} \sum_{h=-\infty}^{\infty} \int_{-\infty}^{\infty} dv_z \int_0^{\infty} 2\pi v_{\perp} dv_{\perp}$$

$$\left[\frac{\omega_{cj}}{v_{\perp}} \frac{\partial f_{jo}}{\partial v_{\perp}} \frac{nJ_n{}^2\left(\frac{k_x v_{\perp}}{\omega_{cj}}\right)}{(-\omega + k_z v_z + n\omega_{cj})} + k_z \frac{\partial f_{jo}}{\partial v_z} \frac{J_n{}^2\left(\frac{k_x v_{\perp}}{\omega_{cj}}\right)}{(-\omega + k_z v_z + n\omega_{cj})} \right] = 0.$$

(2.36)

This form enables one to deduce readily the dispersion relation for longitudinal oscillations in cold magnetized streams.

3. Unstable Longitudinal Oscillations

3.1 Instability in cold streams

The dispersion relation given by equation (2.19) has been applied to such systems as two electron streams in relative motion passing through a stationary positive background, an electron stream passing through an ion stream, and two colliding neutral plasmas for which the interaction of four species of particles has to be considered.

Both PIERCE (1948) and HAEFF (1949) observed fluctuations of long electron beams in electronic devices and interpreted them in terms of space-charge waves, thereby introducing the two-stream instability. PIERCE derived the dispersion relation for an electron stream traveling through a gas of cold ions

$$\frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{(\omega - ku_o)^2} = 1 \quad ,$$

where it is assumed that the wave-vector $\underline{k}$ is parallel to the electron stream velocity u_o. The solution for the wave-vector

$$k = \frac{\omega}{u_o} \pm i \frac{\omega_{pe}}{u_o} \left(\frac{\omega_{pi}^2}{\omega^2} - 1 \right)^{-1/2} \quad ,$$

shows that for $\omega < \omega_{pi}$ there are two types of wave, each having phase velocity equal to the electron stream velocity and having amplitudes which are damped and growing in space, respectively.

HAEFF derived the general dispersion relation (2.19) for a multi-stream system, and applied his result to a single electron stream, in which the oscillations are neither damped nor growing. He also considered a two-stream system for which $\omega_{p1} = \omega_{p2}$, but an algebraic error invalidates his results.

The growth rate in time of longitudinal oscillations in an electron stream traveling through a uniform ion background has been evaluated by

BUNEMAN (1958). If $\omega = |\omega| e^{i\theta}$, the maximum rate of growth in time occurs when $\theta = \pi/3$, and θ takes this value when the resonance condition $ku_o = \omega_{pe}$ is satisfied. The maximum rate of growth is thus

$$[\mathrm{Im}(\omega)]_{max} = \frac{\sqrt{3}}{2}\left(\frac{m_e}{2m_i}\right)^{1/2} \omega_{pe} .$$

For a proton background the shortest e-folding time is $18\omega_{pe}^{-1}$. For a particle background such that $m_1 = m_2$ the e-folding time would be $1.16\omega_{pe}^{-1}$. BERNSTEIN and TREHAN (1960) point out that all perturbations having wavelengths greater than

$$\lambda_c = 2\pi \frac{u_o}{\omega_{pe}}\left[1 + \left(\frac{m_e}{m_i}\right)^{1/3}\right]^{-3/2}$$

are unstable. BUNEMAN suggested that these growing longitudinal oscillations may tend to destroy the electron and ion drifts caused by an external electric field, and may thus provide an electrical resistivity mechanism in a collisionless plasma.

The collision of two identical neutral plasma clouds moving with velocities $\pm u_o$ has been analyzed by KAHN (1957, 1958). The relative motion of the two sets of electrons is converted into longitudinal oscillations within a distance $\lambda \sim \frac{u_o}{\omega_{pe}}$, and subsequently the two sets of ions, moving through the stationary electrons with velocities $\pm u_o$, interact with the electrons to cause additional longitudinal oscillations. The appropriate dispersion relation

$$k^2 = \frac{2\omega_{pe}^{\;2}}{(\omega/k)^2} + \frac{\omega_{pi}^{\;2}}{(\omega/k - u_o)^2} + \frac{\omega_{pi}^{\;2}}{(\omega/k + u_o)^2}$$

has complex roots for ω with $\mathrm{Im}(\omega) > 0$ when $k < k_{max}$. Thus any disturbance with wavelength $\lambda > \lambda_{min}$, where

$$\lambda_{min} = \pi\sqrt{2}\, u_o/\omega_{pe} ,$$

is amplified, and the counterstreaming ion configuration becomes unstable within a distance λ_{min}.

PARKER (1958) also showed that upon the collision of two counterstreaming plasma clouds the electrons are arrested within a few electron plasma periods ω_{pe}^{-1}, while the ions move on. The electrons then form a uniform static electron gas of density $2n_o$ which is perturbed by the ions. PARKER writes the dispersion relation as

$$0 = (\omega^2 - k^2u_o^2)(2\omega_{pe}^2 - \omega^2) + 2\varepsilon\omega^2\omega_{pe}^2(\omega^2 + k^2u_o^2),$$

where $\varepsilon = m_e/m_i$, and shows that complex roots for ω exist when $k^2u_o^2 \leqq 2\omega_{pe}^2$.

If $\varepsilon \ll 1$ and $k^2u_o^2 < 1.6\omega_{pe}^2$, the unstable roots may be expressed as

$$\omega = \pm\, ku_o + \frac{iku_o\varepsilon^{1/2}}{2^{1/2}(1 - k^2u_o^2/2\omega_{pe}^2)^{1/2}} + O(\varepsilon)$$

giving a growth rate of $\exp\left[ku_o t\varepsilon^{1/2}2^{-1/2}(1 - k^2u_o^2/2\omega_{pe}^2)^{-1/2}\right]$. The velocity amplitude of the electron oscillation is larger than that of the ions by a factor of $\left[\dfrac{\varepsilon}{2(1 - k^2u_o^2/2\omega_{pe}^2)}\right]^{1/2}$ and the electrons are therefore accelerated to energies of the order of the initial ion kinetic energy. PARKER suggests this mechanism as a source of high energy electrons.

The growth rate of these oscillations reaches a maximum when $k^2 u_o^2 = 2\omega_{pe}^2$, and the complex roots for ω may be written

$$\omega \cong ku_o \,(0.97 \pm i0.042) \ .$$

PARKER presents a graph of the variation of $\mathrm{Im}(\omega)/\mathrm{Re}(\omega)$ over the entire range $0 \leqq k^2 u_o^2 \leqq 2\omega_{pe}^2$.

DAWSON (1960) investigated the general instability problem of the longitudinal oscillations of a large number of cold electron streams passing through a stationary ion background. He showed that instability always occurs for a finite number of beams, but that a continuous electron velocity distribution, regarded as the limit of an infinite number of beams, is stable. In the limiting process the instability disappears and the results of LANDAU (1946) and VAN KAMPEN (1955) are recovered.

3.2 Instability in warm streams

In Section 2.3, equation (2.25) was obtained as the dispersion relation for warm streams in the absence of particle collisions and an external magnetic field. We now review the effects of temperature on longitudinal oscillations in plasma streams.

J. D. JACKSON (1960) and THOMPSON (1962) give a simple example of the effect of a thermal velocity spread on two-stream flow. Let

$$g(u) = \omega_p^2 \frac{\Delta}{\pi} \left[\frac{1}{(u - U_o)^2 + \Delta_o^2} + \frac{1}{(u + U_o)^2 + \Delta_o^2} \right] ,$$

which represents two identical interpenetrating streams. The dispersion relation has the form of equation (2.26) and has solutions

$$\omega = -ik\Delta_o \pm \omega_p \left\{ 1 + \left(\frac{kU_o}{\omega_p}\right)^2 \pm \left[1 + 4\left(\frac{kU_o}{\omega_p}\right)^2 \right]^{1/2} \right\}^{1/2} .$$

Evidently three of the four solutions are damped in time The fourth is unstable if and only if

$$k^2(U_o^{\,2} + \Delta_o^{\,2})^2 < 2\,\omega_p^{\,2}(U_o^{\,2} - \Delta_o^{\,2}) .$$

Thus growth is only possible if $|U_o| > \Delta_o$. If $|U_o| > \Delta_o$ wave numbers in the range

$$0 < k^2 < \frac{2\omega_p^{\,2}(U_o^{\,2} - \Delta_o^{\,2})}{(U_o^{\,2} + \Delta_o^{\,2})^2}$$

are unstable. The maximum wave number for which growth is possible is $k = \omega_p/2\Delta_o$, and this is reached when $|U_o| = \sqrt{3}\,\Delta_o$. For $|U_o| \gg \Delta_o$, the largest unstable wave number is $k = \sqrt{2}\,\omega_p/U_o$, a result which can be obtained from cold stream theory.

The conditions under which the general dispersion relation (2.25) may have growing solutions have been studied extensively. BERZ (1956) proved that a symmetric single-humped distribution function $g(u)$ is always stable, and AUER (1958) showed that any single-humped distribution function $g(u)$ is stable. J. D. JACKSON (1960) and PENROSE (1960) applied the Nyquist criterion for the instability of servomechanisms to the plasma problem and derived some general results. PENROSE formulated the following stability criterion:

> Exponentially growing modes exist if and only if there is a minimum of $g(u)$ at a value $u = \xi$ such that $\int_{-\infty}^{\infty} (u - \xi)^{-2}[g(u) - g(\xi)]du > 0$. If $g(u)$ has a flat

minimum occupying a finite range $\xi_1 < u < \xi_2$, the instability criterion becomes $\int_{\infty}^{\infty} (u - \xi)^{-2}[g(u) - g(\xi)]du > 0$ throughout the range $\xi_1 < \xi < \xi_2$.

The two-stream instability for cold streams follows immediately. We now consider the two-component plasma in which the distribution functions of both components are Maxwellian, such as an electron/ion plasma or a two-component electron gas in a smoothed-out positive-charge background. This stability problem has been studied by J. D. JACKSON (1960), E. A. JACKSON (1960), PINES and SCHRIEFFER (1961), FRIED and GOULD (1961), and ICHIMARU (1962). The dispersion relation is expressed by equation (2.28) which in general must be solved numerically or graphically, although in some special cases the asymptotic expansions of the function $Z(\zeta)$ can be used. There are an infinite number of solutions, and as the plasma parameters are changed, different roots may become unstable. FRIED and GOULD discuss this point in detail.

The densities of the two components are generally equal in an electron/ion plasma. Problems in the field of semiconductors in which this is not so have been discussed by PINES and SCHRIEFFER. All the above authors except ICHIMARU considered the particular example of a two-component plasma for which $n_1 = n_2$, $T_1 = T_2$, $U_2 = 0$, and showed that growing oscillations are possible only if

$$\frac{\underline{k} \cdot \underline{U}_1}{k} > 1.308\Delta_1 \left[1 + \left(\frac{m_1}{m_2}\right)^{1/2}\right] .$$

For simplicity assume $\underline{k}$ parallel to U_1; the modification for other directions of $\underline{k}$ is obvious. The range of wave numbers $0 \leq k \leq k_t$ for which growth is possible reaches a maximum when

$$U_1 = 2.15\Delta_1 \left[1 + \left(\frac{m_1}{m_2}\right)^{1/2}\right]$$

and then

$$[k_t]_{max} = 0.755\omega_{p1}/\Delta_1 \; .$$

For $U_1 \gg \Delta_1$

$$k_t = \frac{\sqrt{2}\omega_{p1}}{U_1}\left[1 + \left(\frac{m_1}{m_2}\right)^{1/2}\right] .$$

E. A. JACKSON discussed graphical methods of obtaining critical drift velocities and growth rates for $T_1 \neq T_2$. FRIED and GOULD also investigated this problem and gave a graph of the relationship between the critical drift velocity for instability $(U_1)_c$ and T_1/T_2. As the ratio T_1/T_2 increases, $(U_1)_c$ decreases.

ICHIMARU concentrated on a two-component electron gas in a smoothed-out positive-charge background. He considered the range of wave numbers for which growth is possible and in particular the wave number k_c for which growth first becomes possible as the drift velocity is increased, distinguishing between $k_c = 0$ and $k_c \neq 0$.

Thermal velocity spreads also have important effects on the interaction between two neutral plasma clouds. It was pointed out in Section 3.1 that the interaction often occurs in two phases. These have been named Phase I and Phase II by NOERDLINGER (1961). Phase I consists of the initial interaction between the four components and is dominated by the electrons. If Phase I is unstable it is assumed that the two electron streams come to rest at a temperature higher than their initial temperature, while the ions remain unperturbed. Phase II then consists of the interaction between the counterstreaming ions and the stationary electrons.

NOERDLINGER (1960) considered Phase I for the case of two identical Maxwellian streams and obtained results which are identical with those for a two-component plasma already obtained. In 1961 he examined Phase II, assuming that the ion streams are moving with velocities $\pm U_o$, each having density n and temperature T_i, while the electron gas is stationary with density 2n and temperature $(m_e U_o^2/3\kappa)$. The thermal energy of the electrons before the onset of Phase I is neglected during Phase II. NOERDLINGER then showed that instabilities arise whenever the distribution function g(u) possesses a minimum, and that the instabilities are electron/ion oscillations. It is only necessary to consider the interaction of one ion stream with the electrons, and the instability condition is

$$0.011 U_o > \left(\frac{\kappa T_i}{m_i}\right)^{1/2} .$$

A very high stream velocity relative to the ion thermal speed is therefore required. NOERDLINGER also discussed the wavelengths of the growing oscillations.

EK, KAHALAS, and TIDMAN (1962) examined both Phase I and Phase II instability for neutral streams having distribution functions of the form

$$f_{jo}(\underline{v}) = \frac{4n_j \Delta_j^3}{\pi^2[(\underline{v} - \underline{U}_j)^2 + \Delta_j^2]} .$$

As was shown in Section 2.2 the dispersion relation for these functions can be evaluated explicitly. From equation (2.27) the dispersion relation for two identical colliding streams is

$$\frac{1}{\omega_{pe}^{2}} = \sum_{\pm}\left[\frac{(\omega \mp \underline{k}\cdot\underline{U}_o + 3ik\Delta_e)}{(\omega \mp \underline{k}\cdot\underline{U}_o + ik\Delta_e)^3} + \frac{m_e}{m_i}\,\frac{(\omega \mp \underline{k}\cdot\underline{U}_o + 3ik\Delta_i)}{(\omega \mp \underline{k}\cdot\underline{U}_o + ik\Delta_i)^3}\right] .$$

On neglect of the ions, Phase I instability occurs provided that

$$1.46\ |\underline{U}_o| > \Delta_e .$$

Proceeding to Phase II, EK et al. assumed a stationary electron gas of density 2n and mean thermal speed Δ_e', where

$$\Delta_e'^2 = \Delta_e^2 + \frac{1}{3} U_o^2 ,$$

and considered the interaction of one ion stream with the electrons. Phase II instability occurs if

$$\frac{7.05\cdot 10^{-5}}{M} U_o^2 > \Delta_i^2 \left[\frac{\Delta_e'/U_o}{1 + \Delta_e'^2/U_o^2}\right]^3 ,$$

where M is the ion molecular weight.

If Δ_e is neglected, $\Delta_e'^2 = \frac{1}{3} U_o^2$, and the instability condition is

$$\frac{8.6\cdot 10^{-4}}{M} U_o^2 > \Delta_i^2 .$$

The critical drift velocity required for instability has a minimum when $\Delta_e' = U_o$, i.e., $\Delta_e^2 = \frac{2}{3} U_o^2$. EK et al. gave extensive results on growth rates and unstable wave number regions for various values of Δ_e' and Δ_i. Their results agree in general form with those of NOERDLINGER.

KELLOGG and LIEMOHN (1960) considered the interaction of two neutral Maxwellian plasma clouds for a range of relative densities and temperatures. They illustrate graphically the stable and unstable regions as functions of the energy ratios $(\kappa T_1/m_e U_1^2)$, $(\kappa T_2/m_e U_2^2)$ for the density ratios $N_1/N_2 = 1,10$, assuming that $U_2 = 0$. In the extreme case of one plasma at zero temperature, the system is always unstable. For extreme differences between the two plasmas, electron/ion instabilities may occur as rapidly as electron/electron instabilities, and in such cases the separation of the interaction into Phase I and Phase II is invalid.

3.3 Instability in a magnetized plasma

The presence of an external magnetic field modifies the instability results presented in Sections 3.1 and 3.2, and in some instances leads to new effects. The general dispersion relation for longitudinal oscillations is given by equation (2.32), where $\underline{B}_o$ is parallel to the z-axis and $\underline{k}$ lies in the x-z plane. For oscillations parallel to $\underline{B}_o$, $k_x = 0$ and the dispersion relation reduces to

$$H(\underline{k},\omega) \equiv 1 - \frac{1}{k_z^2} \int_{-\infty}^{\infty} \frac{g'(v_z)dv_z}{v_z - \omega/k_z} = 0 \ .$$

Since this is identical with the dispersion relation in the absence of an external magnetic field, the field has no effect on oscillations propagating parallel to it.

SEN (1952) and HARRIS (1959a, b, 1961) have considered longitudinal oscillations with wave vector $\underline{k}$ perpendicular to $\underline{B}_o$ $(k_z = 0)$. For the distribution functions of the form

$$f_{jo}(v_\perp, v_z) = \frac{n_j}{2\pi} \frac{\delta(v_\perp - U_j)}{v_\perp} h(v_z) \ ,$$

which represents particles having a constant speed around the magnetic field lines, the dispersion relation (2.36) becomes

$$1 + \sum_j \frac{\omega_{pj}^2}{2\omega_{cj}^2} \frac{1}{b_j} \frac{d}{db_j} \left[\sum_{n=-\infty}^{\infty} \frac{nJ_n^2(b_j)}{n - \omega/\omega_{cj}} \right] = 0 \ ,$$

where $b_j = \frac{k_x U_j}{|\omega_{cj}|}$. The form of $h(v_z)$ does not appear. SEN determined that in an electron/ion plasma in which the ion motion is neglected, unstable solutions occur for large values of b_j, and HARRIS showed that instability occurs for $b_j > 1.84$.

For oscillations propagating at an angle to $\underline{B}_o$, ($k_x \neq 0$, $k_z \neq 0$), the form of $h(v_z)$ becomes important. HARRIS took

$$h(v_z) = \delta(v_z) \ ,$$

so that the particles have no motion along the field lines, and deduced that in an electron/ion plasma for which the ion motion is neglected, it is possible to find instability for some value of (k_x, k_z) provided that $\omega_{pe} > |\omega_{ce}|$. If the ion motion is included the instability condition becomes $\omega_{pe} > \omega_{ci}$. The fastest growth rate occurs for $k_x \neq 0$ and $k_z \neq 0$.

When the distribution functions f_{jo} possess thermal velocity spreads, two effects may arise. The first is a Landau damping of oscillations for which $k_z \neq 0$. BERNSTEIN (1958) demonstrated that all solutions of the dispersion relation for a stationary isotropic Maxwellian plasma

$$f_{jo}(v_\perp, v_z) = n_j \left(\frac{\gamma_j}{\pi}\right)^{3/2} \exp\left[- \gamma_j (v_z^2 + v_\perp^2)\right]$$

are stable but that Landau damping occurs for longitudinal oscillations with $k_z \neq 0$. If $\underline{k}$ is perpendicular to $\underline{B}_o$, ($k_z = 0$), the oscillations are not damped, but gaps appear in the spectrum of allowed frequencies at multiples of the gyrofrequency.

The second possible temperature effect is an instability caused by temperature anisotropy. HARRIS (1959a, b) chose an electron distribution function

$$f_{jo}(v_{\perp}, v_z) = \frac{n_e \alpha_z}{\pi^2 \alpha_{\perp}^2} \frac{\exp(-v_{\perp}^2/\alpha_{\perp}^2)}{(v_z^2 + \alpha_z^2)} ,$$

neglected the ions and showed that instability only occurs for $k_x \neq 0$, $k_z \neq 0$. As an example, if $k_x = k_z$, $\alpha_{\perp} k_x = |\omega_{ce}|$ and $\alpha_z = 0$, instability occurs if $\omega_{pe} > 1.1 \, |\omega_{ce}|$. HARRIS (1961) stated that a plasma with distribution functions

$$f_{jo}(v_{\perp}, v_z) = \frac{n_o}{\pi^{3/2} \alpha_{\perp}^2 \alpha_z} \exp\left(- \frac{v_{\perp}^2}{\alpha_{\perp}^2} - \frac{v_z^2}{\alpha_z^2}\right)$$

for the electrons and ions possesses unstable longitudinal oscillations for some value of (k_x, k_z) provided that $\omega_{pe} > \omega_{ci}$.

VEDENOV and SAGDEEV (1959) and TIMOFEEV (1961) also studied the effects of temperature anisotropy in electron/ion plasmas. Their ion distribution function is anisotropic, but the form of their electron distribution function is not clear. VEDENOV and SAGDEEV considered wavelengths long compared to the ion gyroradius and obtained a longitudinal instability for $k_x \neq 0$ provided that

$$\frac{8\pi n_o}{B_o^2} \frac{T_{\perp}^2}{T_z} > 1 + \frac{8\pi n_o T_{\perp}}{B_o^2} ,$$

where $\alpha_{\perp}^2 = \frac{m_i}{\kappa T_{\perp}}$, $\alpha_z^2 = \frac{m_i}{\kappa T_z}$. TIMOFEEV considered all wavelengths and obtained instabilities whenever $\alpha_z > \alpha_{\perp}$.

BERNSTEIN and KULSRUD (1960, 1961, 1962) investigated longitudinal oscillations in electron/ion plasmas when $k_x \neq 0$ under the limitation that

$$k_x r \ll 1 ,$$

where r is the gyroradius of an ion at the larger of the ion or the electron mean thermal energies. Under this condition the dispersion relation becomes

$$k^2 = \sum_j \omega_{pj}^{\ 2} \int \frac{F_{jo}'(v_z)dv_z}{v_z - \omega/k_z} ,$$

where

$$F_{jo}(v_z) = \left[2\pi \int_0^\infty f_{jo}(v_\perp, v_z)\ d\ (v_\perp^{\ 2}/2)\right]\left[\int f_{jo}(v_\perp, v_z) d^3v\right]^{-1} .$$

This dispersion relation has the same structure as equation (2.25) for longitudinal oscillations in the absence of an external magnetic field. Consequently a necessary condition for instability is that $\sum_j \omega_{pj}^{\ 2}\ F_{jo}(v_z)$ should possess a minimum.

BERNSTEIN and KULSRUD took various forms for the distribution functions. In their 1960 paper it is shown that if the ions are much colder than the electrons, an electron drift velocity U_e along the field lines causes an instability when

$$U_e > \left(\frac{m_e}{m_i}\right)^{1/2} \Delta_e .$$

If the ions are much warmer than the electrons, instability occurs when

$$U_e > \left(\frac{m_i}{m_e}\right)^{1/2} \Delta_i .$$

In their 1961 paper they considered a Maxwellian ion distribution and an electron distribution

$$f_{oe}(\underline{v}) = f_{oe}(v)\left[1 + \Gamma\varphi(\gamma_e v)\frac{\underline{\varepsilon}\cdot\underline{v}}{\varepsilon v}\right],$$

where $f_{oe}(v)$ is Maxwellian, Γ is a constant, $\underline{\varepsilon}$ represents an electric field applied parallel to $\underline{B}_o$ and $\varphi(x)$ is a function tabulated by SPITZER and HÄRM (1953). In their 1962 paper the electron distribution is Maxwellian and the ion distribution is Maxwellian with a tail modified by the presence of cold neutral molecules.

OZAWA et al. (1962) investigated the general dispersion relation for longitudinal oscillations in a magnetized plasma and obtained a general stability criterion, which is very complicated in contrast to the Penrose criterion for a nonmagnetized plasma. OZAWA et al. applied their criterion to a Maxwellian electron plasma having temperature anisotropy and presented graphs of stable and unstable regions for variations in $\omega_{pe}/|\omega_{ce}|$, $|\omega_{ce}|/\alpha_z k_z$, $\alpha_\perp^2 k_x^2/\omega_{ce}^2$. The plasma is always stable to longitudinal oscillations when $\omega_{pe} < 0.6\ |\omega_{ce}|$.

BUNEMAN (1962) extended his work on the two-stream instability to include an external field. Using the hydromagnetic equations, he analyzed a nonequilibrium situation in which crossed electric and magnetic fields have induced an electron drift across the magnetic field lines while the ions have not had time to respond. By neglecting the external electric field he derived a dispersion relation for longitudinal oscillations

$$\frac{\sin^2\theta}{\omega^2 - \omega_{ce}^2} + \frac{\cos^2\theta}{\omega^2} = \frac{1}{\omega_{pe}^2} - \frac{m_e}{m_i(\omega - \underline{k}\cdot\underline{U}_d)^2},$$

where θ is the angle between $\underline{k}$ and $\underline{B}_o$, and $\underline{U}_d$ is the drift velocity of the ions relative to the electrons. The growth rate of the unstable solutions of this equation is large compared to the rate of change of the ion drift velocity in response to the electric field, provided that the magnetic energy of the plasma is much less than its rest energy. The e-folding time of an instability propagating at the least unstable angle to the magnetic field is proportional to $(m_i)^{1/3}\ (1 + \omega_{ce}{}^2/\omega_{pe}{}^2)^{1/6}$, and a large magnetic field is therefore required to lengthen this time substantially; for large fields only values of θ near 90° are affected. The main effect of the instabilities appears to be a retardation of the electrons rather than an acceleration of the ions, in support of BUNEMAN'S ideas on resistivity in a collisionless plasma.

PARKER (1959) considered the effect of a weak external transverse magnetic field on the collision of two neutral plasma clouds. He neglected temperature effects and assumed that the ions were not affected by the magnetic field. The dispersion relation for Phase II of the interaction is

$$(\omega^2 - k^2 U_o{}^2)^2 \left[\omega^2 - \frac{2\omega_{pe}{}^2\omega(\omega - i\omega_{ce})}{\omega^2 - \omega_{ce}{}^2}\right] - \frac{2m_e}{m_i}\,\omega_{pe}{}^2\omega^2(\omega^2 + k^2 U_o{}^2) = 0 .$$

PARKER showed that unstable solutions exist; the most rapid growth in space occurs when

$$\omega^2 = 2\omega_{pe}{}^2 + \omega_{ce}{}^2 ,$$

and the growth rate in time is of the same order of magnitude as the ion plasma period $\omega_{pi}{}^{-1}$.

3.4 Additional effects

In all preceding calculations we have assumed a collisionless homogeneous plasma and have performed first-order calculations on longitudinal oscillations, neglecting any coupling to transverse modes. A few calculations which have been made without some of these restrictions are now described.

The effect of collisions on plasma oscillations may be included in the hydromagnetic equations by means of a momentum exchange term, so that the equation of conservation of momentum for each species becomes

$$\frac{\partial \underline{u}_j}{\partial t} + (\underline{u}_j \cdot \nabla)\underline{u}_j = \sum_r \frac{\nu_{jr} m_r}{m_j + m_r} (\underline{u}_r - \underline{u}_j) + \frac{q_j}{m_j}\left(\underline{E} + \frac{\underline{u}_j \times \underline{B}}{c}\right),$$

where ν_{jr} is the mean collision frequency of particles of species j with species r.

BUNEMAN (1963) used this collision term in an analysis of longitudinal oscillations in a current-carrying partially ionized gas in an external magnetic field when collisions with neutral particles predominate. His results are discussed in Section 5.2.

In the Boltzmann equation

$$\frac{\partial f_j}{\partial t} + \underline{v} \cdot \frac{\partial f_j}{\partial \underline{r}} + \frac{q_j}{m_j}\left(\underline{E} + \frac{\underline{v} \times \underline{B}}{c}\right) \cdot \frac{\partial f_j}{\partial \underline{v}} = \left(\frac{\partial f_j}{\partial t}\right)_{col.}$$

the choice of a collision term amenable to analysis is more difficult. LEWIS and KELLER (1962) and DOUGHERTY (1963) suggested

$$\left(\frac{\partial f_j}{\partial t}\right)_{col.} = -\nu_j\left(f_1 - \frac{n_1}{n_o} f_o\right)$$

as a suitable first-order term when collisions with neutral particles predominate, and FARLEY (1963a, b) applied this to the same problem as BUNEMAN. FARLEY'S work contains several assumptions about the form of the zero-order distribution functions. His results, applied to an ionospheric problem, are reviewed in Section 5.2.

TIDMAN (1961) considered the effect of small angle Coulomb scattering on two-stream instabilities in a fully ionized gas by means of an expansion of Fokker-Planck collision terms in powers of the collision frequency. He obtained collisional damping of the oscillations which competes with the growth mechanism.

Most plasmas occurring in nature are expected to be nonuniform in space, and the presence of inhomogeneities may modify unstable longitudinal oscillations by causing coupling to transverse modes. Some calculations have been made of the radiation from stable longitudinal oscillations produced by temperature and density gradients and discontinuities, for instance by TIDMAN and WEISS (1961a), but little work has been done on radiation excited by unstable oscillations.

FRIEMAN and PYTTE (1961) studied longitudinal oscillations in an electron plasma with a slightly inhomogeneous distribution function. They assumed that the ions were fixed and used two approaches; the first consisted of a perturbation method

$$f_o(x,v) = f_{oo}(v) + f_{o1}(x,v) ,$$

from which it was shown that small departures from spatial homogeneity may change the stability criteria slightly; the second was a W.K.B. treatment of a slow variation of $f_o(x,v)$ in space from which a stability criterion was obtained.

KRALL and ROSENBLUTH (1962, 1963) found unstable modes for a plasma in a slightly inhomogeneous magnetic field $\underline{B}$ parallel to the z-axis of the form

$$|\underline{B}| = B_o(1 + \varepsilon x) \ .$$

In the 1962 paper only longitudinal modes were considered, but in the 1963 paper low frequency transverse modes were included. A pure transverse mode and coupled longitudinal modes were found to be unstable.

HARRISON (1963) and HARRISON and STRINGER (1963) examined plasmas with a mean velocity $\underline{u}$ parallel to the z-axis, which varies in magnitude in the x-direction. HARRISON considered the "slipping" stream

$$|\underline{u}| = u_o + \alpha x \ ,$$

while HARRISON and STRINGER studied the "adjacent" streams

$$|\underline{u}| = \begin{cases} u_1 & a_1 \leqq x < 0 \\ u_2 & 0 < x \leqq a_2 \end{cases} \ .$$

Conditions for stability of longitudinal modes were established, and thermal velocity spreads were found to increase the stability of the plasmas.

In the absence of an external magnetic field and inhomogeneities, there is nonlinear coupling between longitudinal and transverse waves. STURROCK (1957) used a perturbation analysis of the hydromagnetic equations for a cold stable plasma, and TIDMAN and WEISS (1961b) performed a similar calculation. Two longitudinal waves both at a frequency $\omega = \omega_p$ interact to emit a transverse wave at a frequency $2\omega_p$. STURROCK (1961a) obtained the same result with a Hamiltonian analysis, and showed that the interaction of only longitudinal modes is at least fourth order, whereas the interaction of two longitudinal modes to produce a transverse mode is third order.

As was pointed out in Section 2, linear theory indicates the possibility of growth of longitudinal oscillations, but is insufficient to describe their eventual development. Attempts to solve the nonlinear equations for unstable plasmas have involved numerical analysis or quasi-linear theory.

BUNEMAN (1959, 1961) set up a numerical integration system including nonlinear terms for a cold stream of electrons passing through cold stationary ions. The plasma was treated as one-dimensional, and slight fluctuations were included to initiate oscillations. The results of calculations using 256 particles of each species indicate that the linear theory applied for about three electron plasma periods, after which nonlinear effects became important. After about 50 plasma periods the electron drift was completely destroyed, and at the end of the calculations ($86\ \omega_{pe}^{-1}$) both electrons and ions were in disorder. BUNEMAN concluded that collective effects in collisionless plasmas may act like turbulence in fluid dynamics, redistributing energy among the particles.

The quasi-linear theory has been developed by DRUMMOND and PINES (1962) and by VEDENOV (VEDENOV et al. 1961, 1962; VEDENOV, 1963). It applies to the nonlinear interaction between particles and waves in a weakly turbulent plasma, that is, a plasma in which the energy possessed by longitudinal oscillations is much less than the particle energy but much greater than the thermal equilibrium value of the wave energy. The theory is useful for an unstable plasma if the growth rate of the longitudinal oscillations is much less than their frequency, and it provides the new distribution function in phase space produced by the unstable oscillations. It does not include particle collisions which produce thermal equilibrium on a larger time scale. Recently FRIEMAN et al. (1963) described the Bogoliubov-Krylov technique of multiple time scales by a particular example in which only a single mode is excited, in an attempt to develop a more rigorous basis for the quasi-linear theory.

In Section 3.2 it was shown that thermal velocity spreads impose severe restrictions on the growth of longitudinal oscillations. It has been discovered, however, that many plasmas which are stable to longitudinal oscillations possess unstable transverse modes and the importance of these transverse instabilities is now being recognized.

DAWSON and BERNSTEIN (1958) and HARRIS (1961) demonstrated that a cold stream of electrons moving along the magnetic field through a cold electron/ion plasma excite transverse instabilities. WEIBEL (1959), HARRIS (1961), SAGDEEV and SHAFRANOV (1961), and SUDAN (1963) found transverse instabilities in a plasma with temperature anisotropy, and WEIBEL proved that the presence of an external magnetic field is not essential.

These transverse instabilities have phase velocities much less than the speed of light in contrast to fast transverse waves such as radio waves in a plasma. KAHN (1962) showed that in any electron plasma in which the distribution function has central symmetry, transverse instabilities exist unless $\int_0^\infty U^2 f_o(U,\alpha,\beta)\,dU$, and $\int_0^\infty U f_o(U,\alpha,\beta)\,dU$ are independent of α and β, where (U,α,β) are the spherical polar coordinates of $\underline{v}$. KAHN pointed out that if longitudinal and transverse instabilities occur together in a plasma, the longitudinal instabilities have a greater amplification rate and are therefore more important.

NOERDLINGER (1963) obtained a general dispersion relation for transverse waves propagating parallel to an external magnetic field and considered methods of finding its unstable solutions. He applied his results to an electron gas with temperature anisotropy and to an electron/ion gas with temperature anisotropy, obtaining various transverse instabilities.

BÜNEMANN (1963) found transverse instabilities during the interaction of two identical warm neutral plasma clouds for all values of the cloud velocities $\pm U_o$, whereas, as was shown in Section 3.2, longitudinal instabilities only exist for very large values of U_o.

4. The Physical Mechanisms of Growth

It has been shown in previous sections that in a uniform unbounded collisionless plasma long range Coulomb interaction of the components may give rise to growing oscillations. This may happen when a stream of particles is injected into a plasma, when two neutral plasma clouds collide, when the electrons in a plasma possess a net drift relative to the ions, or when there is some other velocity anisotropy in the plasma. The kinetic energy involved in the anisotropy is converted into oscillatory energy and eventually into random thermal energy. The two physical mechanisms which have been used by various authors to explain these effects are charge bunching and particle trapping.

Charge bunching is a linear mechanism which is most effective in cold streams. If the velocity of a cold stream is slightly perturbed by an oscillatory electrostatic field, a bunching in space of the stream particles occurs and this amplifies the potential of the original disturbance. The space-charge bunching thus sets up growing coherent oscillations and transfers the streaming energy of the particles to the oscillations. Any velocity spread possessed by the stream reduces the coherence of the oscillations, for particles of different velocities gradually move out of phase and phase mixing occurs. A large velocity spread can destroy the collective plasma motion and all oscillations occurring in the plasma are then damped. This is the phenomenon of Landau damping, which was first discovered in a stationary, collisionless Maxwellian plasma by LANDAU (1946). In general, the growth of longitudinal oscillations in a warm stream is maintained provided that the stream velocity is somewhat larger than the mean thermal speed.

Particle trapping is a more complex mechanism and has been the subject of much discussion (BOHM and GROSS, 1949; DAWSON, 1961; KILDAL, 1961, 1963). Particles moving at velocities close to the phase velocity of a wave may exchange energy with the wave and become trapped in the potential wells associated with the wave. Those particles moving slightly faster than the wave give energy to the wave, whereas those moving slightly slower take energy from the wave. Particle trapping may thus cause growth or damping of oscillations. The trapping of a particle cannot occur in less than one wave period, and damping or growth by this mechanism requires several periods to become effective. Since the actual trapping is a nonlinear mechanism it is not responsible for the initial growth of small amplitude longitudinal oscillations, but it does provide an estimate of the maximum energy exchanged between the particles and the wave when the linear approximation is no longer valid (DAWSON, 1961).

5. Natural Phenomena

5.1 Solar radio noise

Solar radio noise is classified into several types; a recent account of all known types has been presented by WILD et al. (1963). Types II and III, consisting of nonthermal radiation in the meter wavelength range, were recorded first by PAYNE-SCOTT et al. (1947) and by WILD and McCREADY (1950). Type II is characterized by bursts of noise which drift from high to low frequencies over the range 200 Mc/s to 10 Mc/s in about 10 minutes. Eighty per cent of the bursts consist of first and second harmonics which have narrow bandwidths, are unpolarized and of comparable intensities (about 10^{-18} watts m^{-2} $(cps)^{-1}$); they both exhibit a small frequency splitting in 85 per cent of the noise signals recorded. The fast drift radio noise designated type III has a drift of about 60 Mc/s per second and a duration of about 2 seconds at 150 Mc/s. Fundamental frequencies observed at 80 Mc/s and 40 Mc/s and the harmonic at 160 Mc/s have relative intensities of about 2:1. The higher fundamental appears to lag the harmonic by some 2 seconds; this delay has been ascribed to a difference in their ray paths.

Chromospheric flares and types II and III radio noise appear to be associated phenomena and, therefore, are considered to have a common origin. Type II bursts tend to appear in the post-maximum phase of the solar flares, while type III can occur in the absence of flares or in groups near the initial stage of the flares. Near visible sun-spot maxima type II noise occurs about once every 50 hours in contrast to type III which occurs at an average rate of 3 per hour.

As some 85 per cent of type II bursts are associated with type III noise, they possess a distinct time relation which suggests that both can originate from a single disturbance. Although it is believed that both types II and III are generated by plasma oscillations, the cause of the plasma instability is less certain. Several authors (e.g., ROBERTS, 1959; MAXWELL and THOMPSON, 1962) have postulated that the compound type bursts arise from growing longitudinal plasma oscillations which are excited by the shock front of a large amplitude disturbance traveling radially outward through the corona and by shock waves propagating at high speeds transverse to it. The former shock induces type II and the latter type III bursts. The position and velocity of the disturbance have been determined from the observed emission frequency and frequency drift rate, respectively, and some model of coronal electron density distribution. The main disturbance is believed to originate at a height of some 5×10^4 km above the photosphere and to move outward along a coronal streamer at a radial velocity of about 10^3 km/sec. The shock waves causing type III bursts move at about 10^5 km/sec. UCHIDA (1962) contends that types II and III radio emissions may have a common radiation mechanism but that their corresponding longitudinal plasma oscillations possess different modes of excitation. While a magnetohydrodynamic shock generates type II bursts, high-velocity electrons streaming outward through strata of diminishing electron density in the corona produce the plasma oscillations responsible for type III bursts. The frequency splitting of type II may represent a gyromagnetic effect analogous to the Zeeman effect. STURROCK (1961b) explained the doublets of the first and second harmonics by the same mechanism since they display a similar sequence. From considerations of the dispersion relations for the magneto-ionic waves he derived a frequency splitting of the order of $1/2(\omega_{ce}{}^2/\omega_{pe})$, a separation which indicates that a magnetic field of 20 G is associated with type II radio noise.

The solar emission was earlier attributed to unstable longitudinal plasma oscillations by MALMFORS (1950) and SEN (1952), who proposed that the electrons in the plasma have a net gyration velocity about the static magnetic field. MALMFORS' dispersion relation contained an error pointed out by GROSS (1951), but SEN obtained the correct dispersion relation and showed that it possessed unstable solutions. He noted that the linear approximation can give only qualitatively significant results.

PARKER (1958) proposed that the plasma oscillations causing type II radio noise are generated by the collision of two uniform interpenetrating neutral plasma streams in what NOERDLINGER (1961) refers to as phase II of the interaction. The growth of plasma instabilities under these conditions was discussed in Section 3, where it was shown that if $2U_o$ is the relative velocity of two identical Maxwellian streams, phase II instability will not occur unless

$$0.011U_o > \left(\frac{\kappa T_i}{m_i}\right)^{1/2} .$$

This condition is not likely to be satisfied in the sun.

In summary, it appears that unstable plasma oscillations are a probable source of types II and III solar radio noise bursts, but that the plasma configuration producing them remains uncertain. To account for the emission of equally intense first and second harmonics, it is necessary to invoke nonlinear in addition to linear mechanisms. STURROCK (1961a) has discussed wave interactions. The conversion of the longitudinal oscillations into radiation may take place at steep density or temperature gradients (TIDMAN and WEISS, 1961a), by local space-charge fluctuations acting as scattering centers (COHEN, 1962) or by magnetic field coupling (BURKHARDT et al., 1961). Nonlinear conversion of longitudinal oscillations to radiation has been investigated by TIDMAN and WEISS (1961b).

5.2 Ionospheric irregularities

The observation of ionospheric irregularities by radio soundings in the 1-4 Mc/s range has recently been supplemented by techniques involving the measurement of backscatter of microwave radiation in the 50-150 Mc/s range. These radar measurements have shown that equatorial sporadic-E irregularities at altitudes of 105 km resemble plane wave-fronts, moving at velocities of about 360 m/sec with wave-normals in a plane normal to the magnetic field lines and wavelengths of 1-3 meters (BOWLES et al., 1963). The irregularities are strongly associated with the flow of current in the equatorial electrojet (COHEN and BOWLES, 1963). Similar irregularities are observed in the auroral regions associated with the auroral electrojets (PETERSON, 1960).

These observations appear to fit an explanation proposed simultaneously by BUNEMAN (1963) and by FARLEY (1963a, b) based on unstable longitudinal plasma oscillations. BUNEMAN'S analysis is hydromagnetic while FARLEY'S is kinetic. The analysis follows the lines of Section 2 and deals with perturbations of a plasma in which the neutrals and ions are stationary and the electrons possess a net drift velocity across the external magnetic field. Collisions of electrons and ions with neutrals play an important part in coupling the equations. The theory assumes that $\nu_e \ll |\omega_{ce}|$ and $\omega_{ci} \ll \nu_i$, which holds for the ionosphere at altitudes in the range 100-110 km. Unstable longitudinal oscillations occur, provided that the electron drift velocity is somewhat greater than the ion thermal speed, a somewhat surprising result as in the absence of collisions and a magnetic field, the electron drift velocity must be greater than the electron thermal speed for instability to occur. When the theory is applied to the electrojet problem in which the electron drift velocity is of the order of 500 m/sec, unstable oscillations occur having wavelengths of 1-3 meters, and phase velocities 405-375 m/sec, propagating within a degree or two of the normal to the magnetic field. There is thus good agreement between experiment and theory.

5.3 Additional observations

The two-stream instability has been suggested as the cause of a number of other natural events which are described briefly:

Auroral electrons. PARKER (1958) suggested that the high energy (50-100 kev) auroral electrons are excited by Phase II instability in the collision of two neutral plasma streams. The energies of the electrons observed in the auroral zone are of the same order of magnitude as the protons in the solar wind, and Phase II instability provides a suitable energy exchange mechanism.

Cometary interactions. HOYLE and HARWIT (1962) considered the possible occurrence of the two-stream instability between cometary particles and the solar wind. They concluded that the observed acceleration of cometary particles cannot be accounted for by this mechanism, but that the solar proton momentum is transferred to cometary ions via a pressure build-up of the transverse magnetic fields imbedded in the solar wind. During periods of enhanced solar activity, however, longitudinal plasma oscillations may occur and may be converted into electromagnetic radiation. Cometary radio noise has never been detected by earthbound equipment, but this does not preclude its existence since any emission would probably be at a frequency below the ionospheric cut-off.

Planetary radiation. Many theories have been invoked to explain the microwave radiation from Venus, and one of these (SCARF, 1963) suggests an instability between the solar wind and the Cytherean ionosphere. Although re-evaluations of the data from Venus favor a thermal source for the radiation (ROBERTS, 1963), a model of solar wind-ionosphere interaction involving plasma instabilities may have some merit for planets possessing only weak magnetic fields.

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